Lecture 21

Elementary Counting Problems (contd.) & Binomial Theorem

Summary of the Last Lecture

Permutations	Linear ordering of $[n]$	$n!$
	Linear ordering of multisets	$\dfrac{n!}{a_1!.a_2!....a_k!}$
Sequences	k-length sequences from $[n]$	n^k
	k-length sequences from $[n]$ where elements cannot repeat	$\dfrac{n!}{(n-k)!}$
Subsets	k-element subsets of $[n]$	$\binom{n}{k}$
	k-element multisubsets of $[n]$	$\binom{n+k-1}{k}$

Examples

Example: A medical student has to work in a hospital for 5 days in January. However, she is not allowed to work two consecutive days in the hospital. In how many different ways can she choose the five days he will work in the hospital?

Solution: It's like selecting a 5 element subset from $[31]$ such that no two elements are consecutive.

$A =$ Set of 5 element subsets from $[27]$

$B =$ Set of 5 element subsets from $[31]$ s.t. no two elements are consecutive

Define a bijection $f : A \rightarrow B$

$$f(\{x_1, x_2, x_3, x_4, x_5\}) = \{x_1,\ x_2 + 1, x_3 + 2, x_4 + 3, x_5 + 4\}, \text{ where } x_i < x_{i+1}.$$

Prove it a bijection yourself.

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Examples

Example: A company has 20 employees, 12 males and 8 females. How many ways are there to form a committee of 5 employees that contains at least 1 male and at least 1 female?

Solution:

$$12 \times 8 \times \binom{18}{3}$$ ✗

Select a male → 12

Select a female → 8

Select the rest of the 3 people. → $\binom{18}{3}$

$$\binom{20}{5} - \binom{12}{5} - \binom{8}{5}$$ ✓

 # committees of 5 → $\binom{20}{5}$

 # committees of 5 without a female → $\binom{12}{5}$

 # committees of 5 without a male → $\binom{8}{5}$

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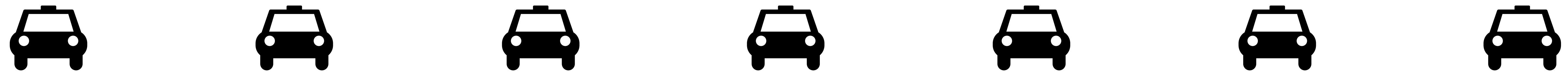
Examples

Example: A car dealership employs 5 salesman. A salesman receives a ₹ 1000 bonus for each car he sells. Yesterday the dealership sold 7 cars. In how many different ways can this happen? *(Two scenarios are different if they result in distinct bonus payments.)*

Solution:



Multiple Scenarios

1	1	2	2	5	4	1
2	3	1	4	1	5	1
2	1	1	4	5	2	1
3	1	3	4	2	2	4
5	5	5	4	4	4	4

Same

The number of different scenarios is the number of 7-element multisubsets of $[5]$. ∎

Two Tips

- If counting “good object” is not easy, then count the total number of “objects” and subtract the number of “bad objects” from it.
- If finding the size of a set, say A, is not easy, then find a set, say B, whose size you already know and give a bijection between A and B.